



What is an estimate of measurement uncertainty?

An estimate of measurement uncertainty is a value that expresses the uncertainty or reliability of a given measurement value. When the conditions affecting a measurement result in a less certain measurement value, the uncertainty value will be higher. When the conditions affecting a measurement result in a greater certainty in the measurement value, the uncertainty value will be lower. The uncertainty value will be expressed in the same dimension and with the same units as the measurement value. For example, if the measurement being described by the uncertainty value is a length measurement expressed as a number of millimeters, then the uncertainty value will also be a length expressed in millimeters. (Uncertainty can be expressed in units different from those of the measured value if the units are related by some factor. For example, if a measurement value is expressed in pounds, the uncertainty of the measurement might be expressed in ounces.) Though the rules and formulas for calculating an estimate of measurement uncertainty can be quite specific, it is important to remember that the calculated value is just an estimate. There are a number of ways to reach the estimated value, and the estimated value is not a hard bound of the possible values of the measured parameter. If one were to express measurement uncertainty using informal terms, those terms might include “probably” or “most likely”, but not terms like “certainly” or “definitely”.

Why estimate measurement uncertainty?

There are several reasons for calculating measurement uncertainty. At the very least, the list of reasons would include: to evaluate the quality of a given measurement to determine if the measurement can be improved, to communicate the quality of a measurement to those that will use a measurement value so that they can better understand values that rely on the measurement value supplied with the uncertainty value, to determine whether artifacts subject to a measurement are likely to meet tolerances despite errors in measurement, and to advertise the measurers’ capabilities as a calibration laboratory might do on a scope of accreditation. If the measurer takes the time to calculate an estimate of measurement uncertainty, he or she will also find which conditions affecting the measurement contribute the most to measurement uncertainty, and may be able to reduce the effects of those conditions resulting in a higher quality (lower uncertainty) measurement. When a measurement value is supplied to the user of the value along with an uncertainty value, the measurement value will be worth more to the user. A familiar example of this is the measurement of calibration artifacts like ring gages, gauge blocks, plug gages, and pins. Imagine if, for example, a gauge block of nominal size 4 inches was supplied to a user with no associated uncertainty value. The user could use the 4 inch gauge block to measure parts or to calibrate other gauges, but would have no idea how far those measurements and calibrations are from the actual accepted length of 4 inches. If, on the other hand, the 4 inch gauge block was supplied with a measurement value and an associated measurement uncertainty, the user could estimate an uncertainty for his or her measurements that expresses the quality of those measurements. Similarly, the user can only determine if a part or gauge is within tolerance if he or she knows the uncertainty of the reference used in the measurement of the part or gauge. Imagine if someone wants to determine if a ring gage is within the XX ring gage

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tolerances. This person uses an appropriate measurement device to compare the size of the ring gage under test to a master ring gage. Imagine that both the ring gage under test and the master ring gage read at exactly the nominal value of the two gages. Now imagine that the uncertainty of the master gage is ten times the tolerance of a XX ring gage. In that case the measurement outcome is basically meaningless – though the master ring gage’s value reads nominal, its actual size could be very far from nominal. The measurement value for the measured ring gage would then also be very far from nominal. If on the other hand, the measurement value of the master ring gage was believed to be within a few micro-inches of the nominal value, then the measurement value of the ring gage under test might also be within a few micro-inches of nominal (of course this would depend on other conditions of the measurement situation besides the uncertainty of the measurement of the master ring gage). Finally, measurement uncertainty values can be communicated to measurement customers so that they can determine if a measurement suppliers capabilities will meet their needs. If, for example, I need to determine if the pin gauges that I am measuring are within 0.000010 inches of their nominal size, I might need a master that is likely to be within 0.000005 inches of its certificate value. To find a suitable master reference I want not only an artifact of consistent geometry, but also someone to calibrate the artifact with a measurement uncertainty that is lower than 0.000005 inches so that I can be relatively confident that the ping gauges that I measure are within the specified tolerance. To find that supplier I will review potential suppliers’ scopes of accreditation to determine if they can provide a measurement of the artifact with low enough uncertainty.

How widely can a value be applied?

An estimate of measurement uncertainty is specific to one measurement or range of measurements. The users of a measurement instrument often look for an uncertainty value that covers all of the measurements made with that instrument. Rarely is it possible to use one value as the estimate of measurement uncertainty for all of the measurements made with a given measurement instrument, especially when users usually want to find the lowest reasonable estimate of measurement uncertainty for a given measurement. Variables that will lead to differences between the measurement uncertainty of one measurement and another – even if made with the same measurement instrument – would include: the uncertainty of measurement for the master gauges used for the measurement, the environmental stability of the measurement environment, the contact geometry and the geometry of the masters and artifacts under test, the sizes of masters and artifacts under test, the materials of the masters and the artifacts under test, among others. Pratt & Whitney’s Labmaster Universal is an excellent example of a measurement instrument that can be used in myriad applications – each with its own unique measurement uncertainty due to the factors listed above. For example, if one were to use the LMU 175 to measure a 1-inch gage block using bidirectional probes, using a 1 inch gage block calibrated by Starrett-Webber with an uncertainty of 0.000003 inches as a master, and performed the measurement in a laboratory with temperature stability of $\pm 0.5^{\circ}\text{F}$, one might estimate the measurement uncertainty at about 0.000004 inches. If, on the other hand, the LMU 175 were used to measure a 3 inch ring gage, the estimated measurement uncertainty, subject to all of the same variables, would likely be higher, like 0.000012 inches. For most measurements made on those high precision measurement instruments, the largest source of uncertainty is likely the uncertainty of the

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masters. If the masters are calibrated by a national metrology lab, then the measurement uncertainty is probably as low as it can be. Any measurement is a comparison between one unknown parameter and some known reference or unit. Besides a naturally occurring intrinsic standard – a calibrated artifact is the best form of this known reference. Now, though each measurement will have its own measurement uncertainty, those that advertise their measurement capabilities will often express the measurement uncertainty with a linear expression that relates the uncertainty of the measurement type to the size of the artifact that is measured or to the position of the measurement probe. For example, Pratt & Whitney Measurement Systems reports the uncertainty of our calibration of ball bars on our A2LA scope of accreditation with a linear expression that accommodates all of the lengths of ball bar that we are capable of calibrating: $(12 + 0.78L) \mu\text{in}$ where L is the length of the ball bar that we calibrate. This linear expression is an estimate equal to the equation of best fit line for pairs of values relating the length of a ball bar to the uncertainty of the measurement of the ball bar.

How is measurement uncertainty expressed?

The uncertainty value will be expressed in the same dimension and with the same units as the measurement value. For example, if the measurement being described by the uncertainty value is a length measurement expressed as a number of millimeters, then the uncertainty value will also be a length expressed in millimeters (or a related unit that is some multiple or factor of millimeters such as microns or nanometers). The uncertainty will be expressed as a value that bounds a range of values centered on the measurement value. Usually, the expression will use the plus-minus symbol, “±”, followed by the uncertainty value. For example, if the measurement of a ring gage is reported as 3.000001 inches ± 5 micro-inches, this means that the diameter of the ring gage is likely between 2.999996 and 3.000006 inches, and most likely is 3.000001 inches. Typically, when the measurement uncertainty is expressed along with a measurement value, it will also include the notation “@ 95% C.L., K=2”. This part of the expression means that the chance that the true value is not within the range indicated by the value following the plus-minus sign is only 5%. “k” is the number that indicates the coverage factor. Typically metrologists will deal with coverage factors of 1, 2, or 3. An uncertainty expressed at k=2 will be double the uncertainty expressed at k=1. An uncertainty expressed at k=1 indicates that the true value is 68% likely to fall within the band defined by the ±value. An uncertainty expressed at k=3 will be triple the uncertainty expressed at k=1. An uncertainty expressed at k=3 indicates that the true value is 99.7% likely to fall within the band defined by the ±value.

What are some ways to estimate measurement uncertainty?

Generally, an uncertainty value will be calculated through a root-sum-squares calculation. This means that values for the magnitude of the effect of uncertainty sources are squared before being summed together. The final uncertainty value (the standard uncertainty expressed at k=1) will be the square root of this sum. The uncertainty sources and their expected magnitudes can be organized in an equation or organized in an uncertainty budget. Pratt & Whitney publishes an equation that gives a coarse estimate of the uncertainty of measurements made with the LabMaster Universal instruments. The equation is published in Appendix A of the instruments’ users’ manuals. The equation entails the same calculation described above (root-sum-squares), but includes only the ‘heavy hitters’, the most significant sources of uncertainty. For most measurements made with any of the LabMasters Universal,

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the most significant source of uncertainty will be the uncertainty of the masters used to set the instrument scale and to set the instrument datum. The equation in Appendix A calls for the uncertainty values of the masters used to set instrument scale and to set the measurement datum, and includes a smaller value to cover the expected contribution of the Labmaster's function in an acceptable environment. The outcome of the equation will not be as good a representation of the measurement uncertainty as a calculation based on an uncertainty budget, but the value will be close to that found with an uncertainty budget. An uncertainty budget is created in three steps. First, one lists all of the possible contributors to uncertainty that may affect the measurement. Next, one must determine the possible impact of each contributor, and give it a value in terms of the measured parameter. Finally, all of those values are combined by root-sum-square calculation. The value found through these calculations is referred to as the standard uncertainty. Typically the standard uncertainty value will be doubled so that the uncertainty can be expressed as the 'expanded uncertainty' – in the form: $\pm X @ k=2$ (95% C.L.), where X is the expanded uncertainty value.

What are the typical sources of uncertainty that should be listed in an uncertainty budget?

An uncertainty budget should include any and all of the uncertainty sources that can contribute significant error or uncertainty to a measurement. For most measurements, that will include sources arising from the measurement equipment used, master references, the measurement environment, and the unit under test. Some sources of uncertainty (a single line item listed on the budget) will be a function of sources acting in tandem. In other words, the value of one line item accounts for the effect of uncertainty arising from multiple sources. For example, repeatability of measurement of a pin or thread wire between flat probes may be a function of the geometry of the pin or wire (straightness and roundness, the unit under test), the parallelism of the flat probes (the measurement instrument), the resolution of the measurement instrument (the measurement instrument), and the stability of the measurement environment (measurement environment). Other line items will be more or less isolated from others. For example, in an uncertainty budget for the measurement of pins or thread wires, anvil parallelism would likely be its own line item. When constructing an uncertainty budget, one should try not to count any sources more than once, but there are some sources that must be included per published standards, and some of those sources are affected by others that may be counted in their own line item. Any uncertainty budget should include at least: Repeatability of the measurement, Resolution of the measurement instrument, resolution of the unit under test, the reproducibility of the measurement, the uncertainty of datum and scale masters, and uncertainty values that arise from environmental instability.

About the author

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